line and its performance checked the curves presented almost exactly. The curves presented on the rat race check very well with published data.[6]

Again it must be emphasized that pure shunt junctions are assumed neglecting fringing effects. This will only be true if the ratio of wavelength to line size is very high. This first approximation is valuable anyway. To make a coaxial 3-db directional coupler with many arms as shown in Fig. 11 requires excessively small center conductors since the characteristic admittance varies as the logarithm of the size ratio. But with waveguide a branch guide 3-db coupler, with many arms appears to be a definite possibility since the impedance of the arms varies as the height of the guide and the fringing effects get smaller as the height of the guide is decreased.

[6] H. T. Budenbom, "Some quasi-biconjugate networks and related topics," Proc. of the Symposium on Modern Network Synthesis, Polytech. Inst. of Brooklyn, New York, N. Y., 1952, pp. 312–326.

Broad-Band Waveguide Series *T* for Switching*

J. W. E. GRIEMSMANN† AND G. S. KASAI‡

Summary—**By use of properly proportioned half-wavelength transformer sections in the arms of a waveguide series *T* broad-band performance can be obtained for switching applications. Over the frequency band 8200 to 9765 megacycles per second, corresponding to a bandwidth of 17.4 per cent, an experimental model showed an insertion vswr of less than 1.15 for transmission through the aligned arms and 1.30 for transmission around the bend. Further bandwidth improvement is possible with the use of a special arrangement of quarter-wave transformer sections but at the expense of further impairment of power-carrying capacity.**

Introduction

ONE APPLICATION of a *T* junction is duplexing. The transmitter and antenna are usually connected to the aligned arms and the receiver to the side arm. Power flow from transmitter to antenna and not to the receiver can be considered to be so directed by an appropriately positioned effective short in the side arm. The received signal can be considered to be directed from antenna to side arm by an appropriately positioned effective short in the transmitter arm. An inherent bandwidth limitation for the ordinary *T* is that the effective short positions are correct only for the center frequency and thus give rise to "branching loss,"[1] principally through reflection of energy back out through the antenna. The device discussed in this paper provides one means for minimizing this branching loss over a broader range of frequencies.

In any practical case of duplexer design many special problems relating to positioning of the shorts and special means of solving them are introduced.[1] This paper is directed chiefly at the frequency sensitivity of the stubbed *T* structure itself and means for overcoming it. Fixed metallic shorts are assumed to replace the effective shorts indicated above. For this reason the device is called a switching *T* rather than duplexer.

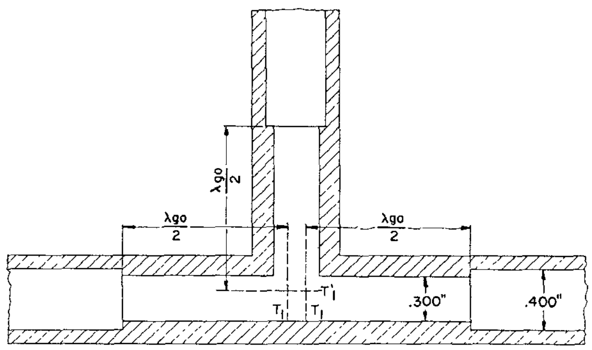


Fig. 1—Waveguide switching *T* cross section—side view.

Shown in Fig. 1 is a dimensioned sketch of the broad-banded series (*E* plane) *T* in the *X* band, RG-52/U rectangular waveguide designed for a center frequency of 9000 mc and a theoretical bandwidth of 16 per cent for a maximum vswr of 1.08. This design was based on circuital computation of a pure series arrangement of the arms and use of the equivalent circuit[2] for the *E* plane *T* at 9000 megacycles per second. At appropriate reference planes the particular equivalent circuit chosen is a good approximation to a series con-

* Manuscript received by the PGMTT, July 16, 1956. Presented before the National Symposium on Microwave Techniques, Philadelphia, Pa., February 2–3, 1956. This work was performed under Signal Corps Contract No. DA-36-039-sc-42489. The work is in part the subject of a thesis for the M.E.E. degree by Mr. Kasai at the Polytech. Inst. of Brooklyn.

† Microwave Res. Inst., Polytech. Inst. of Brooklyn, Brooklyn, N. Y.

‡ North American Aviation Corp., Downey, Calif.; formerly with Microwave Res. Inst., Polytech. Inst. of Brooklyn, Brooklyn, N. Y.

[1] L. Smullin and C. Montgomery, "Microwave Duplexers," Rad. Lab. Ser., McGraw-Hill Book Co., Inc., New York, N. Y., vol. 14, ch. 7, 1948.

[2] N. Marcuvitz, "Waveguide Handbook," Rad. Lab. Ser., McGraw-Hill Book Co., Inc., vol. 10, pp. 337–351, 1951.

nection of the arms, and was thus assumed for the design. The discontinuity susceptances of the steps were compensated for by foreshortening of the transformer sections.[3]

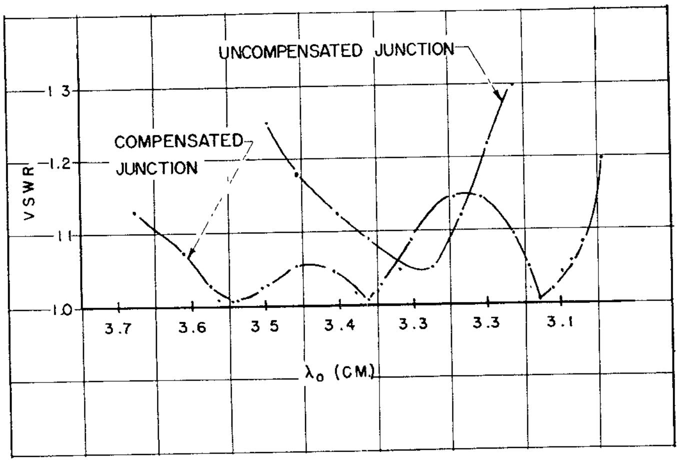


Fig. 2—Experimental results: insertion vswr vs free space wavelength for transmission between parallel arms.

Shown in Fig. 2 is a plot of measured insertion vswr vs free space wavelength for transmission through the aligned arms of the broad-band T along with the same for an uncompensated T. At a vswr of 1.15 the bandwidth for the broadband T is measured to be 19.1 per cent whereas that for the uncompensated T is 6.2 per cent.

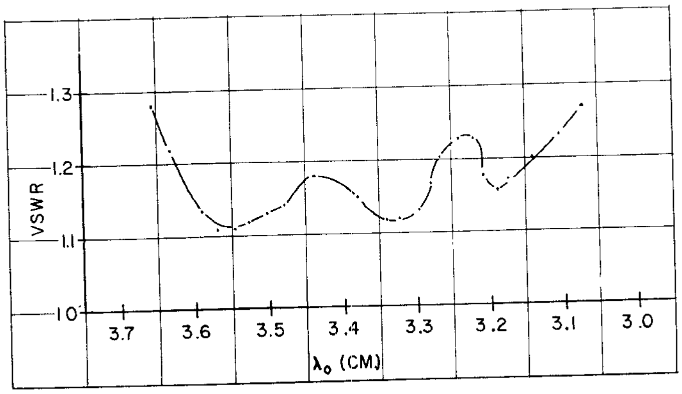


Fig. 3—Experimental results: insertion vswr vs free space wavelength for "around the corner" transmission of the compensated junction.

Shown in Fig. 3 is a plot of measured insertion vswr vs free space wavelength for transmission around the bend of the broad-banded T. For a vswr of 1.23 the bandwidth is found to be 15.8 per cent while for a vswr of 1.29 the measured bandwidth is in excess of 17.4 per cent. The minimum insertion vswr value that could be measured on the uncompensated T junction for around the bend transmission at 9.0 kmc was 1.30.

The degree of correspondence between theoretical

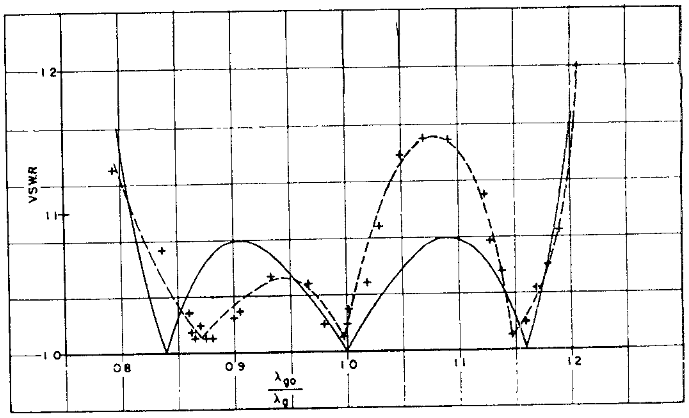
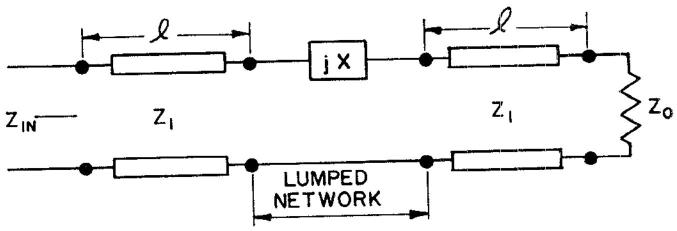


Fig. 4—Comparison of experimental and theoretical results for the compensated junction: – – – =experimental; —— =theoretical.

and measured vswr data is shown in Fig. 4 for transmission through the aligned arms. Considering the magnitude of the scales used and the fact that the model represents an initial paper design the agreement is good. For the transmission around the bend the same theoretical data would be valid and the discrepancy is beyond that anticipated from the equivalent circuit. Further improvement in performance could be expected if additional investigations were conducted, particularly with respect to compensation for the around the bend transmission.

General Circuit Analysis

The following analysis assumes that the T may be represented by the simple circuit consisting of the three arms connected in series.



$$\frac{X_r}{Z_1} = \frac{2\left[\left(\frac{Z_o}{Z_1}\right)^2 - 1\right] \tan \beta\ell}{1 + \left(\frac{Z_o}{Z_1}\right)^2 \tan^2 \beta\ell}$$

$$\frac{dX_r}{d(\beta\ell)} = \frac{2\left[\left(\frac{Z_o}{Z_1}\right)^2 - 1\right] \sec^2 \beta\ell \left[1 - \left(\frac{Z_o}{Z_1}\right)^2 \tan^2 \beta\ell\right]}{\left[1 + \left(\frac{Z_o}{Z_1}\right)^2 \tan^2 \beta\ell\right]^2}$$

Fig. 5.

In Fig. 5 is shown the circuit used for the analysis when there is but a single transformer in each leg. The circuit is somewhat more general than need be for design and is used to determine the values of the series reactance, x_r, required to match the over-all structure.

[3] *Ibid.*, pp. 307–310.

By appropriate analysis of the half structure,[4,5] the value of x_r shown in Fig. 5 was determined. In accordance with Foster's reactance theorem, the expression for the derivative of x_r with respect to frequency as given in Fig. 5 is positive for $(z_0/z_1) > 1$ and for values of l/λ_g ranging less than $\frac{1}{8}$ on either side of $l/\lambda_g = \frac{1}{2}$, corresponding to an approximate limit of the guide wavelength ratio of 5 to 3 (per cent λ_g bandwidth = 50). For small values of tan Bl the reactance x_r can be approximated by a shorted arm or stub having a length equal to that of the transformer section. A match can be obtained at two other frequencies by appropriate choice of the characteristic impedance of the stub.

Analysis for Half-Wave Transformer Case

If the T is to be used as a switching junction with equal transmission for the transmit and receive condition, the stub must also be made up of a line of characteristic impedance, Z_1, of length, ι, and shorted at the end of this length. For this condition, the input reflection factor is

$$|\Gamma| = \sin \tan^{-1}\left[\frac{\tan \beta l}{\frac{z_0}{z_1}(1 + \tan^2 \beta l)}\left(\frac{3}{2} - \left[\frac{z_0}{z_1}\right]^2 \cdot \left[1 - \frac{1}{2}\tan^2 BL\right]\right)\right]$$

The junction is matched for $\Gamma = 0$. This occurs for $\tan \beta l = \tan 2\pi l/\lambda g_0 = 0$ corresponding to $l/\lambda g_0 = \frac{1}{2}$ and for

$$\tan \beta l = \tan \frac{2\pi l}{\lambda g_0} = \pm \left| \left[2 - 3\left(\frac{z_1}{z_0}\right)^2\right]^{1/2} \right|.$$

Thus, there are three frequencies for which $\Gamma = 0$. Shown in Fig. 6 the curve is the ratio (z_0/z_1) plotted as a function of $\lambda g_0/\lambda g_c$, where λg_c is taken to correspond to the highest frequency of reflection cancellation. Between midband, corresponding to λg_0, and the cancellation frequencies the reflection coefficient and corresponding insertion

$$\text{vswr} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

go through a maximum value. Plotted as curve B in Fig. 7 is this maximum vswr as function of $\lambda g_0/\lambda g_c$, giving one measure of bandwidth. Shown for comparison are the maximum vswr values measured at λg_c for the uncompensated junction ($z_1 = z_0$). Typically for a vswr of 1.08 and a design centered at 9.0 kmc in RG-52/U guide, the per cent frequency bandwidth for the compensated junction is 16 per cent as compared to 2.5 per cent for the uncompensated junction. The value of z_0/z_1 to give this condition is 1.329. The basic circuit is seen to be capable of considerable broadbanding as compared to that of the uncompensated with, of course, the attendant decrease in power carrying capacity associated with the reduction in height of the guide, the standing waves in the transformer section, and the corner of the discontinuity.

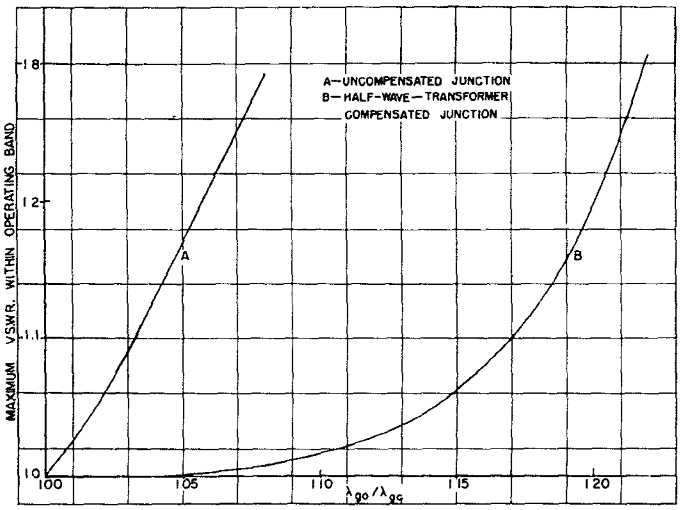


Fig. 7—Maximum insertion vswr vs cancellation wavelength.

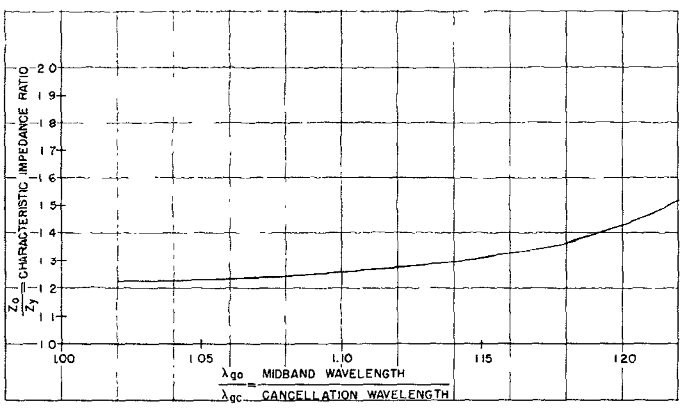


Fig. 6—Transformer characteristic impedance ratio vs cancellation wavelength.

Double Quarter-Wave Case

Further improvement in broadbanding is feasible if instead of one-half wave transformer in each arm, two quarter-wave transformers are used in each arm as shown in Fig. 8. The analysis can again be accomplished in terms of the open circuit and short circuit half structures, but is considerably more complicated.

Shown in Table I are the maximum vswr values occurring at frequencies between the matched condition at the center frequency corresponding to λg_0 and the matched conditions at the frequencies corresponding

[4] J. W. E. Griemsmann, "Microwave Broadbanding," Proc. of the Symposium on Modern Network Synthesis, Polytech. Inst. of Brooklyn, Brooklyn, N. Y., pp. 330–331, April, 1952.

[5] H. Bode, "Network Analysis and Feedback Amplifier Design," D. Van Nostrand and Co., Inc., pp. 363–364; 1945.

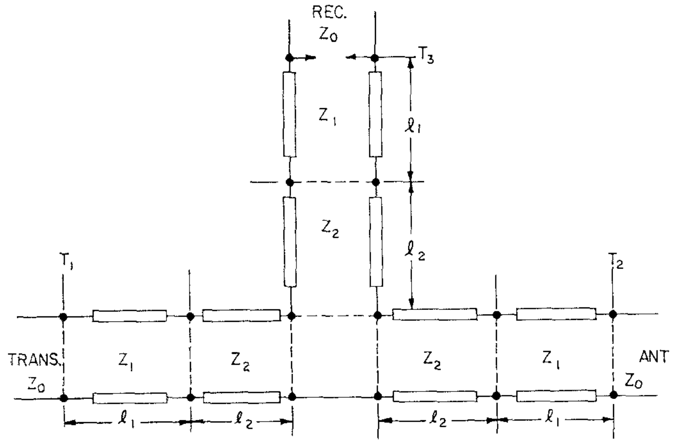


Fig. 8—Double quarter-wave transformer circuit.

to λg_c for given values of $\lambda g_0/\lambda g_c$ (where λg_c corresponds to the highest cancellation frequency) and various values of the transformer impedance ratio Z_1/Z_2. The analysis shows that for a given selection of $\lambda g_0/\lambda g_c$ there is a relationship between z_1/z_2 and z_0/z_1 given implicitly by

$$(2b - 3a^2)y^4 + (6a^2b - 3b^2 - 2b + 3)y^2 + (2b - 3a^2b^2) = 0$$

where

$$a = \frac{z_1}{z_0}, b = \frac{z_1}{z_2}, \text{ and } y = \cot\frac{\beta l}{2}.$$

TABLE I
DOUBLE QUARTER-WAVE PERFORMANCE CHARACTERISTICS
(CALCULATED VALUES)

$\lambda_{g0}/\lambda_{gc}$	1.16	1.20	1.25
z_1/z_2	Maximum Insertion VSWR		
0.300	1.0212	1.2121	
0.500	1.0456	1.1386	1.7682
0.700	1.0586	1.1530	1.5899
0.800	1.0651	1.1649	1.5956
1.000	1.0786	1.1935	1.6519
1.100	1.0854	1.2095	1.7139
1.200	1.0926	1.2266	1.7657

For each value of vswr given in the table there is also a value of z_1/z_0 inferred.

The table shows that for a given bandwidth the maximum vswr values may be reduced below the value for the half-wave transformer junction, $z_1/z_2 = 1.00$, by using the double quarter wave arrangement. For example for $\lambda g_0/\lambda g_c = 1.16$ corresponding to that for the previously considered half-wave transformer design, the maximum vswr value could be reduced from 1.079 to 1.046 using $z_1/z_2 = 0.500$ and $z_0/z_1 = 0.9044$. For $\lambda g_0/\lambda g_c = 1.20$ and $\lambda g_0/\lambda g_c = 1.25$, the values for the maximum vswr values are noted to pass through minimums as the transformer dimensions are varied. The upper bound for bandwidth is changed very little, if any, over that of the half-wave transformer.

In order to achieve the improvement indicated above for $\lambda g_0/\lambda g_c = 1.16$, the height of the guide in the second transformer section would have to be considerably increased over that corresponding to z_0. This brings into doubt the practicability of the double transformer arrangement particularly in view of the degree of improvement obtained. From the design viewpoint, the question of higher modes and discontinuities would make the realization difficult. Reducing the value of z_0 below that of standard RG-52/U guide by transformers or tapers so as to minimize the mode and discontinuity problems would mean considerable complication of the design and reduction in carrying capacity.

COMMENTS

For some switching purposes T's can be broadbanded up to approximately a limit of 16 per cent. This is not an open recommendation for use of the broad-banded switching T as the basis for radar duplexing. There is competition from short slot duplexer which has about the same bandwidth, and in some cases can provide better magnetron starting characteristics.

Some of those having long acquaintance with the microwave art will recognize this as analogous to the broadbanding of stub supports for coaxial lines. It will be recalled, however, that these used a parallel circuit for the stub and quarter-wave transformers sections. This paper is felt to give the first open suggestion for use of this device in rectangular waveguides and to give the first thorough bandwidth analysis for the half-wave transformer case.